

Geometric Aspects of Strategic Putting

John F. Mahoney and Daniel P. Connaughton

University of Florida

Wonseok (Eric) Jang

Texas Tech University

Putts comprise the plurality of strokes in a round of golf. This research studies putting using imagery, physics, and results taken from the golf literature. Putts are restricted to balls rolling on a horizontal uniform surface. The hole is assumed to be virtual with the success of putts (successful putts are those which would be captured by the hole if it were real) being indicated by their stopping points. The collection of all such stopping points may be encapsulated thus producing a region of success. The mathematical representation of the perimeter of this region is developed. The speed of the putting surface as given by the Stimp meter reading is a parameter in the formulation. Particularly desirable stopping points within the region of success are indicated.

Keywords: putting, imagery, physics

In what has become known as “regulation” golf a score of par on a hole consists of two portions. The initial portion involves the number of strokes required by a hypothetical golfer to advance the ball from the tee to the green. This number is taken to be two fewer than par since it is presumed that the final portion consists of two additional strokes which are in the form of putts. Hence, in an idealized round of regulation golf, 36 strokes are attributable to putts. Even though golf professionals and other skilled players regularly reduce the number of putts per round to less than 30 it is proper to assert that putting is the most prevalent shot in golf (Karlsen, Smith, & Nilsson, 2008). For this reason we provide new and detailed insight that is intended to lead to an understanding of improved putting.

This study ultimately concerns the putting of a golf ball on a uniform horizontal surface. The target is a regulation golf hole. The origin of the putts is a point sufficiently far from the target so that all successful putts enter the hole on paths which are essentially parallel. The focus is on successful putts.

Mahoney is with the Department of Industrial and Systems Engineering University of Florida, Gainesville, FL. Connaughton is with the Department of Tourism, Recreation & Sport Management, University of Florida, Gainesville, FL. Jang is with the Department of Media and Communication, Texas Tech University, Lubbock, TX. Address author correspondence to Daniel Connaughton at danc@hhp.ufl.edu.

A different perspective on successful putting is now explored. Under the conditions stated above consider the collection of all successful putts which originate from the same point. Suppose through deception the hole was not real, but was an optical illusion. Putts which were destined to fall into the hole would continue to roll until they stopped of their own accord. Let the stopping points of all of these potentially successful putts be noted. These stopping points could then be enclosed in a tightly drawn mathematical envelope. An appreciation of this envelope provides the golfer with an alternative intention. Rather than simply aiming at the hole the intent now could be to cause the ball to stop within the visualized envelope. A greater understanding of this virtual envelope will be explored.

Many psychologists and golf professionals (e.g., instructors and players) have identified mental imagery as one of the most effective technique for enhancing quality of training programs (e.g., skill acquisition) and athletic performance (Driskell, Copper, & Moran, 1994). Essentially, mental imagery is defined as “the process of imaging the performance of a skill with no related overt actions” (Cohn, 1990, p. 304). Many golfers use imagery to enhance their performance in the golf field (Taylor & Shaw, 2002). For example, before golfers make the actual putt, they might imagine themselves hitting the ball at 17-inches beyond the hole and subsequently the ball just reaching that imaginary point. Existing research has shown that, when a golfer uses imagery before he or she hits the ball, the golfer performed better in various types of shots such as approaches (Brouziyne & Molinaro, 2005) and putting (Ramsey, Cumming, & Edwards, 2008; Taylor & Shaw, 2002). The positive effect of imagery on performance applies to all levels of athletes—that is, novice and expert golfers are able to enhance their performance by engaging in imagery (Bernier & Fournier, 2010). Nevertheless, despite this universal benefit, research has found that successful and higher-level athletes more frequently engaged in imagery than unsuccessful and lower-level athletes (Callow & Hardy, 2001; Cumming & Hall, 2002; Hardy, Jones, & Gould, 1996).

The notion of aiming at a point beyond the hole is not new. The saying “never up, never in” implies the concept. Dave Pelz specializes in the teaching of the fine points of golf’s short game (Pelz, 2000). For years he has advocated aiming at a point 17-inches beyond the hole, however, Pelz’s 17-inches rule is not universally accepted (Tempoman, 2009).

The seminal work on the interaction of a golf ball with the hole is set forth by Holmes (1991) and further explored by Penner (2002).

Qualitative Considerations

Consider a regulation golf ball rolling on a uniform horizontal surface as it encounters a regulation golf hole. If the ball’s speed is not too great, it will be captured by the hole. Balls which are rolling too fast will jump over the hole and continue rolling beyond the hole. There is a critical ball velocity at the initial rim of the hole, call it v_c , such that larger values result in the ball jumping over the hole, while smaller values cause the ball to be captured by the hole. The value of v_c depends upon the portion of the hole that is hit. The largest value of v_c is called V and occurs for balls striking the hole at its widest point.

It is now imagined that the hole does not actually exist, but that it only appears to be present. Putts directed toward this virtual image continue to roll until their kinetic energy is completely spent owing to the friction between the ball and the surface. There is an important boundary curve which can be outlined on the putting surface which separates the stopping points of putts which would have been captured by the hole (if one had been there) from the stopping point of putts which would have jumped over the hole. This boundary curve is given as the locus of stopping points of putts which have the velocity of v_c at the initial rim of the hole.

The so-called speed of the putting surface is measured with the aid of a Stimpmer (Holmes, 1986, USGA, 2012). This is a ramp-like device which provide a golf ball with a standard initial rolling speed of v_2 . The resultant roll on a horizontal portion of the putting surface is measured in feet and is denoted by S . S values typically range from a value of four for very slow surface up to 14 for exceedingly fast surfaces.

The symbolic notation which is used is summarized and reinforced in Table 1.

Quantitative Considerations

Figure 1 represents the geometry of the hole as seen from above. A rectangular coordinate system is in place with its origin at the center of the hole. Putts are struck in the positive y direction, and are assumed to originate at relatively large negative values of y . Putts hit the hole at values of x in the range $-R \leq x \leq R$, where R is the radius of the hole. Owing to the symmetry of the situation only the mathematical consequences of the right half of the figure need be explored—the left half is a mere reflection of the right half. Recall, that in this study the hole only appears to exist. The success or failure of a putt is determined by its stopping point.

Table 1 Nomenclature

k	= proportionality constant, $[\theta^2 / L]$
R	= radius of hole, $[L]$
S	= Stimpmer reading, $[L]$
v	= ball velocity, $[L / \theta]$
v_c	= critical ball velocity at hole, $[L / \theta]$
v_2	= ball velocity produced by Stimpmer, $[L / \theta]$
V	= minimum velocity of ball required to bridge diameter of hole, $[L / \theta]$
x	= coordinate variable perpendicular to path of ball, $[L]$
y	= coordinate variable parallel to path of ball, $[L]$
Δy	= distance traveled by ball whose initial velocity is v , $[L]$
λ	= conversion factor for length, $[1]$
ξ	= normalized version of x , $[1]$

Though not essential it is convenient to introduce ξ which is the normalized substitute for x ,

$$\xi = x / R \quad (1)$$

This expresses x as a fraction of R so that as x varies from zero to R , ξ varies from zero to one.

Successful putts must surely reach the initial rim of the hole. The formula for the rim of the hole is

$$x^2 + y^2 = R^2 \quad (2)$$

From this

$$y^2 = R^2 - x^2 = R^2 [1 - \xi^2] \quad (3)$$

And it follows that the initial rim is given by

$$y = -R\sqrt{1 - \xi^2} \quad (4)$$

or in dimensionless terms

$$y / R = -\sqrt{1 - \xi^2} \quad (5)$$

Energy consideration lead to the notion that the amount of roll a golf ball experiences on a uniform level surface is directly proportional to its kinetic energy. Since the kinetic energy of a rolling ball is proportional to the square of its velocity it follows that

$$\Delta y = kv^2 \quad (6)$$

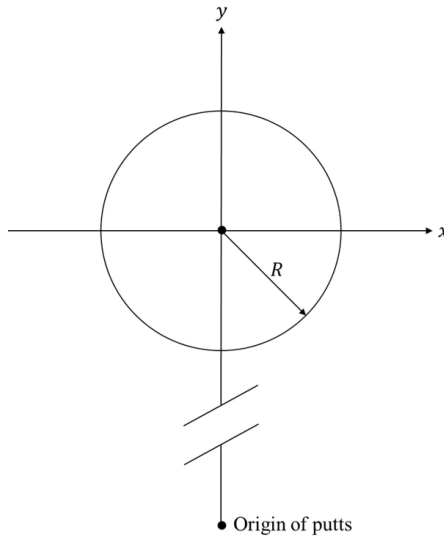


Figure 1 — Circular hole as seen from above. The hole has a radius R and diameter D . The origin of the $x - y$ coordinate system is at the center of the hole. Putts travel in the positive y direction and hit the hole for values of x , $-R \leq x \leq R$.

where Δy is the length of roll and v is the ball's velocity. The value of the proportionality "constant" k depends upon the speed of the surface and upon the units adopted for Δy and v . Stimp meter readings may be employed to establish the value of k . The Stimp meter produces an initial ball velocity of v_2 , while the extent of the roll is reported in feet as S . Hence, upon substituting into (6) one gets

$$S\lambda = kv_2^2 \quad (7)$$

from which k may be found. Since S is always reported in feet, λ is introduced as a conversion factor so that the roll may be presented in equivalent measures of length.

Recall that the boundary which separates the stopping points of potentially successful putts from putts which are hit too hard is determined by the locus of stopping points which have the velocity v_c at the initial rim of the hole. From (6), the amount of roll such putts experience is

$$\Delta y = kv_c^2 \quad (8)$$

Penner (2002, eq. 23) shows that

$$v_c = V(1 - \xi^2) \quad (9)$$

hence

$$\Delta y = kV^2(1 - \xi^2)^2 \quad (10)$$

Eliminating k between (7) and (10) gives

$$\Delta y = (S\lambda)(V / v_2)^2(1 - \xi^2)^2 \quad (11)$$

Subsequent division by R produces

$$\Delta y / R = S(\lambda / R)(V / v_2)^2(1 - \xi^2)^2 \quad (12)$$

According to the USGA (2011, p. 27), the diameter of the hole must be 4.25 inches. This means that R is 2.125 inches. Since R is expressed in inches this requires λ to equal 12 since this is the factor used to convert S from feet to inches. In an analysis of the Stimp meter, Holmes (1986) finds that v_2 is 1.83 m/sec. Holmes (1991) also gives V as 1.626 m/sec. Inserting these four numerical values into (12) gives

$$\Delta y / R = 4.46S(1 - \xi^2)^2 \quad (13)$$

The stopping points of putts which determine the boundary curve is obtained by the addition of (5) and (13). The result is

$$y / R = 4.46S(1 - \xi^2)^2 - \sqrt{1 - \xi^2} \quad (14)$$

Over much of the range of ξ the second term on the right side of (14) is insignificant compared with the first term.

Shape of the Envelope

The envelope consists of two parts. The lower part serves as a front door to putts. It is described by the semicircular trace given by (5). All potentially successful putts must at least get to that boundary. Successful putts which cross that boundary are

further constrained in that they must not cross the next boundary which is given by (14). Hence (5) and (14) encapsulate the region of success.

Visualization of the envelope given by (5) and (14) may be accomplished in a number of ways. There are computer programs which produce the plot directly. Alternatively, another program (or even a hand-held calculator) may be used to generate the data points which may then be plotted manually. This is labor-intensive since the data set for (14) depends upon the value of S selected. Table 2 provides the data as generated by (14) for values of S of 5 and 10. In addition, the table gives data for plotting (5), although the result may also be drawn using a compass. Henceforth, no further consideration need be paid to the plotting of (5).

Table 2 *Values of y / R used to Plot (14), for S Equal 5 and 10, as well as y / R Values Used to Plot (5)*

ξ	y / R from (14) ($S = 5$)	y / R from (14) ($S = 10$)	y / R from (5)
0	21.30	43.60	-1.000
.05	21.19	43.38	-0.9987
.10	20.82	42.72	-0.9950
.15	20.25	41.63	-0.9887
.20	19.57	40.12	-0.9798
.25	18.63	38.23	-0.9682
.30	17.51	35.98	-0.9539
.35	16.23	33.41	-0.9367
.40	14.82	30.55	-0.9165
.45	13.29	27.77	-0.8930
.50	11.68	24.22	-0.8660
.55	10.01	20.86	-0.8352
.60	8.33	17.47	-0.8000
.65	6.68	14.11	-0.7599
.70	5.09	10.89	-0.7141
.75	3.61	7.88	-0.6614
.80	2.29	5.18	-0.6000
.85	1.19	2.91	-0.5668
.90	0.3691	1.1742	-0.4359
.91	0.2443	0.9033	-0.4146
.92	0.1342	0.6603	-0.3919
.93	0.03946	0.4465	-0.3676
.94	-0.03903	0.2631	-0.3412
.95	-0.1003	0.1117	-0.3122
.96	-0.1429	-0.005862	-0.2800
.97	-0.1652	-0.08733	-0.2431
.98	-0.1640	-0.1291	-0.1990
.99	-0.1322	-0.1234	-0.1411
1.00	0.0000	0.0000	0.0000

Another approach to understanding how expressions are pictured is covered in the early stage of a calculus course. In that context note that ξ^2 is the independent variable of both (5) and (14). This means that both functions are what is known as even functions. That is, their values are indifferent as to the sign of ξ . This implies symmetry with respect to the y / R axis. This has been noted before and means that it is sufficient to focus solely on the right half of their plots—the left half is merely a reflection of the right half.

It is easily shown that (14) cuts the ξ - axis in two places. Surely $\xi = 1$ is one such location. The other location is at ξ_1 which depends on the value of S . Table 3 gives values of ξ_1 for a range of S values.

Calculus may be invoked to find the location of the maximum and minimum of (14). The maximum occurs at $\xi = 0$ where the value of y / R is $(4.46)S - 1$. The minimum depends upon S and is located at ξ_2 . Values of ξ_2 along with the corresponding value of y / R —here called $(y / R)_2$ —are listed in Table 3 for values of S .

The foregoing serve to produce the sketch of (5) and (14) shown in Figure 2. The figure is not drawn to scale but illustrates the critical information. The figure seems to agree with many people’s preconceived notion of how the envelope should appear—except for the downward bulge at ξ_2 . The downward bulge occurs at values of ξ quite close to one, and as such qualifies as an edge effect. It is unclear whether this edge effect is real or if it is merely the consequence of applying an inexact mathematical expression in that region. The interaction of the ball with the rim is more complex near the edge of the hole. For putts which are centrally directed, the ball is either captured or it jumps over the hole. Putts near the edge of the hole may be subject to a more complicated mechanism since they have been observed to roll around the far rim before capture or escape. As previously mentioned, the second term of (14) is usually insignificant as compared with the first term, but it is this second term which leads to the curious bulge. When (5) and (14) are drawn to scale, as in Figure 3, it is seen that the downward bulge is not noticeable and so it is of no practical consequence.

Table 3 Zeroes and Local Minima for (14)

S	ξ_1	ξ_2	$(y / R)_2$
4	0.9239	0.9705	-0.1808
5	0.9348	0.9746	-0.1679
6	0.9425	0.9776	-0.1580
7	0.9482	0.9798	-0.1500
8	0.9528	0.9815	-0.1435
9	0.9564	0.9829	-0.1380
10	0.9594	0.9841	-0.1332
11	0.9620	0.9851	-0.1291
12	0.9642	0.9859	-0.1254
13	0.9660	0.9867	-0.1221
14	0.9677	0.9873	-0.1191

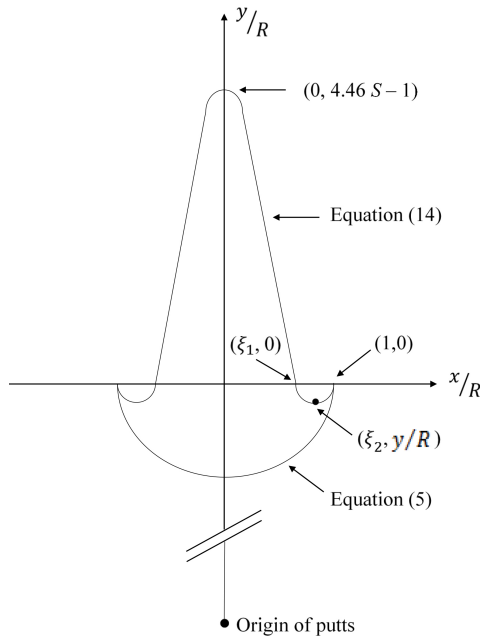


Figure 2 — Sketch of equations (5) and (14), not drawn to scale.

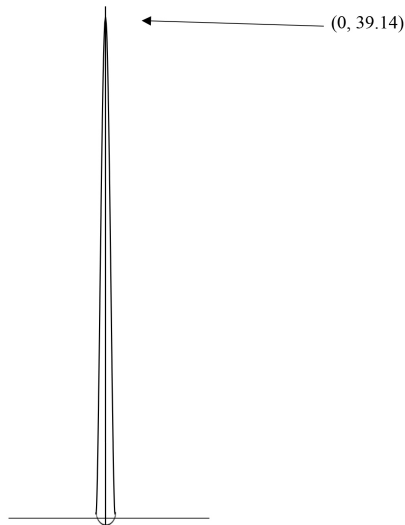


Figure 3 — Pictured is the plot of (5) and (14) drawn to scale for $S = 9$. The ordinate is y/R and the abscissa is ξ . Note that the downward bulges which are prominent in Figure 2 are so small that they do not appear when drawn to scale.

Strategy

Pelz (2000) has suggested that golfers visualize a point 17-inches beyond the far side of the hole and then hit the putt with sufficient pace to just reach the visualized point. This 17-inch rule (henceforth called Rule 1) is unrefined since it does not take into account the speed of the green. Rules 2 and 3 as suggested by Tempoman (2009) address this perceived problem. Rule 2 is said to be for conservative putts and places the aim point S inches beyond the hole, where S is the Stimpmeter reading. The more aggressive Rule 3 puts the aim point $2S$ inches beyond of the far side of the hole. Rules 4 and 5 are variations of Rule 3. Rule 4 has the aim point $2S$ inches beyond the center of the hole. Rule 5 is the same as Rule 4 except now the measurement starts at the initial side of the hole.

There may not be a single way to choose among these five rules. One way is to examine the character of putts which travel the distance advocated by each method. Then enter (14) with the appropriate data and solve for ξ . Although not emphasized previously, ξ has the important significance in that it represents the fraction of the holes' diameter which is available for a successful putt. From one perspective larger values of ξ are desired, but at the same time larger values of ξ imply smaller values of y/R . Smaller values of y/R promote the likelihood that putts will not be struck firmly enough to reach the hole. This means that the resultant values of ξ should be on the large side, but not so large that they are a result of y/R values which are small enough to promote the possibility of having putts fall short of the hole. In addition, the resultant value of ξ should be relatively constant across the range of S values.

Table 4 gives the spectrum of ξ values for each of the five rules. Scrutiny of this table is not likely to result in selecting one rule above all of the others. Rule 1 has an appealing 17-inch distance parameter but the ξ values vary greatly from top to bottom in the table. The ξ values for Rule 2 are acceptably uniform and are pleasing owing to their ample size, but golfers may be challenged by having the aim point only S inches beyond the hole. Rules 3, 4, and 5 form a progression in

Table 4 Values of ξ for the Five Aiming Rules

S	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5
4	0.507	0.744	0.667	0.707	0.750
5	0.580	0.759	0.680	0.713	0.747
6	0.629	0.769	0.689	0.716	0.745
7	0.664	0.777	0.696	0.719	0.743
8	0.691	0.782	0.701	0.721	0.742
9	0.713	0.786	0.705	0.723	0.741
10	0.731	0.790	0.708	0.724	0.741
11	0.746	0.793	0.710	0.725	0.740
12	0.759	0.795	0.712	0.726	0.740
13	0.770	0.797	0.714	0.727	0.740
14	0.779	0.799	0.716	0.727	0.739

that they differ only in that the aim point is retarded by R inches as Rule 4 is compared with Rule 3, and as Rule 5 is compared with Rule 4. All three rules result in ξ values which are reasonably level.

Focusing on Rule 3, 4, and 5 one strategy might be to accept Rule 4 as a compromise of all three. Another strategy would be to examine the notion of a region to be aimed at rather than aiming at a single point. Let Rule 5 provide the near boundary for the region by placing it transverse to the axis of the putt at a distance $(2S - R)$ inches from the center of the hole. This boundary would extend ξR inches on either side of the axis. The ξ value called for is for Rule 5 as read from Table 4. In a similar fashion Rule 3 provides the far boundary of the region. In particular this boundary is located transverse to the axis at a distance $(2S + R)$ inches from the center of the hole. It also extends ξR inches on either side of the axis, but this time ξ is specific for Rule 3. Longitudinal segments are now added to enclose the region. The region is almost a rectangle, but owing to the two different ξ values used in its construction the figure is actually an isosceles trapezoid.

Alternative Targets

The benefit of attempting to have putts stop on a virtual target strategically placed beyond the hole is to reinforce proper alignment but also to encourage proper ball speed in the vicinity of the hole. The target shaped like an isosceles trapezoid that we found has appealing theoretical properties but from a practical perspective other shapes may be easier to envision and may serve the golfer even better.

One variation on using the isosceles trapezoid is to slightly distort that figure into a rectangle of like dimensions. A virtual rectangle four inches long and three inches wide placed $2S$ inches center to center from the hole would serve.

Perhaps the simplest substitute target to grasp is gained by having the golfer take a mental picture of the hole and then translating it $2S$ inches downstream. The resultant image is slightly too wide, but this minor infraction is more than compensated for by the ease of visualization.

Summary

Students of putting are asked to imagine that the hole is covered over with turf and that the success or failure of a putt is determined by the stopping point of the putt. Analysis based upon plausible assumptions leads to the conclusion that only putts which stop in the region enclosed by (5) and (14) are capable of success. This region is not static but depends upon the speed of the green as measured by S , the Stimpmeter reading.

A serious problem of putting is that of falling short of the hole. Unfortunately, the solution to this problem is not to merely hit putts much harder. The reason for this has to do with the shape of the region of success which continues to narrow as putts are hit further. The situation is somewhat like trying to drive successfully when a long fairway narrows to a point. It is clear that the golfer's intention must be the result of compromise. The ball must be struck with enough strength that it gets to the hole, but must not be struck so firmly as to encounter excessive narrowing of the success region.

There are minor variations of the compromise strategy that we suggest. Perhaps the easiest to grasp involves taking a mental picture of the hole and then translating it 25 inches downstream where it is mentally attached. As usual, the actual hole is assumed to be paved over with sod. The golfer's objective is to make putts stop on the translated picture.

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