

# Haptic Perception-Action Coupling Manifold of Effective Golf Swing

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Researchers have used full-body models to investigate how the dynamics of the golf swing, including those related to wrist function, may be affected by the rotation of individual anatomical segments. Recent studies have suggested that the inertia tensor, a physical property whose values are time- and coordinate- independent, may be an important informational invariant haptic perception of which is used to control limb movement. We mechanically characterized the perception-action coupling manifold, which seems to be a more parsimonious representation of performance of the golf swing. We used this 'dynamic touch' concept to explain why much of traditional golf coaching has stressed the importance of a neutral left wrist position. We tested the hypothesis that the aforesaid 'flat left wrist' has an important influence on the efficacy of the golf swing by comparing the perception-action coupling manifold of both a high- and a low- handicap player. We found that perception and action were more highly correlated with each other in the more skilled player. The implications of our preliminary findings about wobbling inertia for functioning of the 'flat left wrist' are discussed.

**Keywords:** haptic perception; perception-action coupling manifold; instantaneous screw axis (ISA); wobbling inertia

Researchers have reported on the kinematics and kinetics of the golf swing in terms of the relative contribution of each of the anatomical structures that are involved (Horan, Evans, Morris, & Kavanagh, 2010; S. MacKenzie & Sprigings, 2009a, 2009b; Nesbit, 2005; Sprigings & Mackenzie, 2002; K. Teu, Kim, Tan, & Fuss, 2005; K. K. Teu, Kim, Fuss, & Tan, 2006). Here, we asked whether, rather than using the customary full-body computer model, the mechanics of the golf swing could be modeled in terms of a more parsimonious integration of perception and action.

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Human movement control can be seen as a process that is distributed over the performer-environment system i.e., not something that is isolated to either the performer or his/her environment. The performer and his/her environment may be said to be coparticipants in any resulting action. The argument, then, is that behavior is intrinsically functional, rather than the combination of intrinsically mechanical and only extrinsically or secondarily functional (Reed, 1985). In this way, actions are specific to function rather than to mechanism. Movements and postures are controlled and coordinated to realize functionally specific acts that are themselves based on the perception of affordances (i.e., possibilities for action [Gibson, 1979]). The “what” and “where” of control of a certain action is, from this ecological perspective, less amenable to expression in anatomical terms, and more easily conceptualized in terms of the interrelational properties of specific task constraints.

For example, whenever a person wields a grasped object such as a golf club, the forces that are produced vary over time. These forces bring about motions of both the body tissues and of the grasped object, and these motions also vary over time. The fact that perception is used to control this movement implies that the changing forces and motions must be linked by some property that does not change as they do. Thus, the required time-invariant properties have to extract an identity from the object that is being wielded (e.g., “this is a long, thin object”) that is independent of the details related to that object that are varying over time (e.g., regardless of whether the person swings the object through long arcs or short jerks). Seminal work conducted by Solomon and Turvey (Solomon & Turvey, 1988; Solomon, Turvey, & Burton, 1989) has suggested that the haptic perception of length during effortful or dynamic touch is related to the inertia tensor of the object that is being wielded. The term haptic perception refers to the perception that is itself based on feedback from the mechano-receptive machinery that is embedded in the body’s deformable tissues. Haptic perception is perception by means of the body, in concert with the general definition of perceptual systems (Gibson, 1966, 1979). It functions in two distinct ways, each of which may act either in isolation or concurrently: a) proprioceptively—perception of the body and perception of the body’s segments relative to the body as a unit and relative to each other—and; b) exteroceptively—perception of attachments to the body (e.g., handheld objects) and of surfaces adjacent to the body (Turvey, 2007). The haptic system participates both in perception and in action, as is apparent in the case of dynamic touch (see Appendix C for details).

It has been suggested that the collected information variables that are relevant to perception and action in a given task can be represented on a low-dimensional simple form or manifold (Harrison, Hajnal, Lopresti-Goodman, Isenhower, & Kinsella-Shaw, 2011; Warren, 2006). One form of representing the club’s motion is the Instantaneous Screw Axes (ISA) (Kiat Teu & Kim, 2006; Kim & Kohles, 2011) (see also Appendix A). That is, the motion of the club is a synthesis of the whole body action of the performer. However, action is guided by perception (Gibson, 1979; Warren, 2006). In this case, the performer haptically perceives the time invariant variable of the club (inertia), over time, to control his or her action on the club.

We used principal axes of inertia ( $e_3$ ) to represent the perceived inertia tensor, following the work of Carello and Turvey (2004). Moreover, we used ISA and  $e_3$  instantaneous successive correspondence, for generating a manifold for the perception-action coupling during the swing. This coupling forms a synergy. In

this way, the golfer produces a larger kinetic energy when the club is permitted to naturally select its own ISA (the instantaneous screw in the action manifold) from the abundant degrees of freedom that the space around the performer allows, than when he/she arbitrarily restricts, or creates, arbitrary screws for the system. We expect that a skilled golfer would be more attuned to the instantaneous inertia axes,  $e_3$ , permitting a smoother ISA.

In fact we expect that any disturbance in alignments of the two axes may be due to abnormal wrist extensor/flexor movement during downswing. This kind of wrist action may perturb the coupling of  $e_3$  and ISA at any instant. This is why the coaching literature has such an emphasis on the need for a “flat left wrist” (Jenkins, 2007). A “non-flat left wrist” may create wobbling inertia. Wobbling inertia is essentially a quick oscillation of a relevant part of a body in motion (Fisher, 1970). Initially, the rest of the body remains mostly unaffected, until the oscillation is translated into a body yaw of increasing amplitude and, consequently, a loss of control. A practical example of this phenomenon is the balancing of an automobile tire so that it does not wobble, which basically entails adjustment of the distribution of mass such that the principal axis of inertia of the wheel is aligned with the axle.

Following an ecological approach to the  $e_3$  and ISA relationship, the first aim of this study was to define how a performer controls a full-body action golf swing by perceiving the club’s inertia when said club is being acted upon. We essentially attempted a mechanical characterization of the perception-action coupling via the spatial (free- and line-) vector components (see Appendix B) for such time invariant properties. We then applied both the free-vector and line-vector eigenvalue problem to the spatial inertia tensor of a wielded golf club.

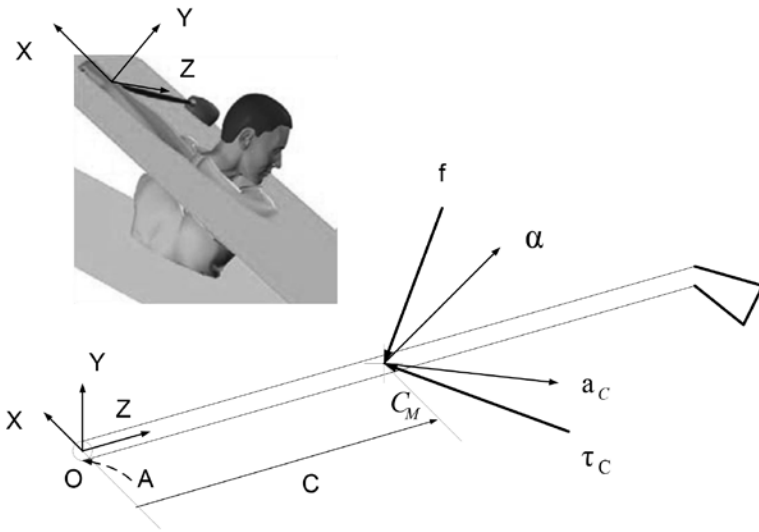
The second aim of our study was to apply the perception-action coupling manifold to contrast how an expert and a novice player move across the information manifold of perception and action to produce a golf swing (Lagarde, Kelso, Peham, & Licka, 2005). Our working hypothesis was that our more skilled golfer would exhibit a closer correspondence between  $e_3$  and ISA, in line with his/her better attunement to the inertia tensor of the golf club.

## Method

### Screw Representation of Line-Vector, Free-Vector, and the Spatial Inertia Tensor

We developed a representation of the spatial vector quantities that are relevant to the action of the golf swing, through the use of screw theory (Ball, 1998) to signify a directed line in space. Instantaneous Screw Axes (ISA) are fundamental to the description of the motion of a rigid body on a specific environment. The instantaneous motion of the club may be considered to rotate about the ISA. It is important that the general motion of the performer’s action toward his/her perceived orientation of the hand-held club be such that it can be characterized by the ISA of the club. ISA is considered to be the sum of contributions of all the joints and muscles that are involved during the golf swing (and not only those that are influenced by wrist action). Line coordinates (or Plücker Coordinates) (Hunt, 1990) were used to describe screw motion (i.e., rotation with translation) and thus to locate screw surfaces in terms of the set of ISA (see Appendix A for details).

The linear map from the spatial acceleration space to the spatial force space is defined as the spatial inertia tensor in this study (Figure 1 and Appendix A). Since the dynamic touch paradigm is based on the detection of deviation from (normally) established patterns (Craig & Bourdin, 2002), it is thought to be analogous to the error covariance tensor that is encountered in estimation theory for linear systems (Rodriguez, 1987). The equivalence between estimation and dynamics in the sense of the dynamic paradigm is outlined in Table 1. Note that the spatial inertia tensor summarizes the inertia and mass properties of a golf club about the wrist

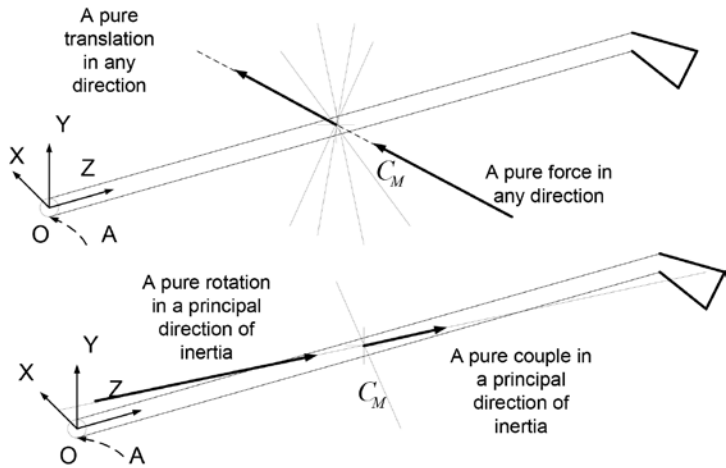


Due to its special 6×6 form, the screw inertial tensor at ‘O’

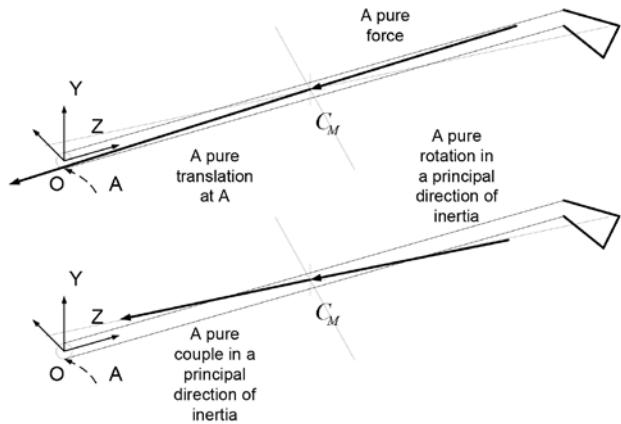
$$M_o = \begin{pmatrix} mI & H \\ H^T & I \end{pmatrix} = \begin{bmatrix} 0.788 & 0 & 0 & 0 & -29.241 & -1.001 \\ 0 & 0.788 & 0 & 29.241 & 0 & 1.108 \\ 0 & 0 & 0.788 & 1.001 & -1.108 & 0 \\ 0 & 29.241 & 1.001 & 2855.908 & -5.206 & 123.719 \\ -29.241 & 0 & -1.108 & -5.206 & 2859.116 & 111.823 \\ -1.001 & 1.108 & 0 & 123.719 & 111.823 & 20.966 \end{bmatrix}$$

**Figure 1** The grip reference frame was attached to the center of the end of the club shaft. The spatial inertia tensor  $M_o$  was computed with reference to the origin of the grip coordinate system. Its decomposition matrix contains the principal axes and moment of inertias, represented at “O”. The same is true for the decomposition at a point along, for example, the wrist joint axis, “A”. That is, no matter where the problem is posed the same eigen vectors/values form the basis of the decomposition, but represented at “A”. The pure force-translation direction is aligned neither to the z-axis nor to the principal direction of inertia. The eigenvalues of  $I$  (or principal moment of inertias,  $I_1$ ,  $I_2$ , and  $I_3$ , where  $I_1 \leq I_2 \leq I_3$ ) are the club’s resistance to rotation about the respective directions of the eigenvectors (or principal axes of inertias,  $e_3, e_2, e_1$ ). The original values of quantities as well as the anatomical sketch that were originally published by (S. MacKenzie & Spriggins, 2009a) are used by permission of professor Sasho MacKenzie and are represented relative to the grip reference frame. (units in  $\text{kg} \cdot \text{cm}^2$ ).

joint. The system can then be described more simply by separating the quantities into constitutive and geometric contents (Ciblak, 1998). This results in a family of problems collectively known as eigenvalue problems. An alternative and useful eigenvalue problem for the analysis of the dynamic touch paradigm was proposed in our paper. Therefore, we proceed with a method of free-vector and line-vector decomposition of the spatial rigid-body inertia tensor (see Appendix B for details).



**Figure 2** Eigentwist and corresponding eigenwrench of the inertia tensor at the center of mass ( $C_M$ ). Since the eigenscrews pass through  $C_M$ , they are also the principal screws of the inertia tensor.



**Figure 3** Eigentwist and corresponding eigenwrench of the inertia tensor at a point away from the center of mass ( $C_M$ ). Since the eigenscrew is centered at a point  $A \neq C_M$ , the pure force through  $A$  only causes a pure translation.

**Table 1    Equivalence in Optimal Estimation and Robot Dynamics.**

Estimation		Robot Dynamics
States	$\begin{bmatrix} f \\ \tau_c \end{bmatrix}$	Spatial force
Costates	$\begin{bmatrix} a_c \\ \alpha \end{bmatrix}$	Spatial acceleration
Transfer matrix	$\Phi$	Spatial Jacobian
Process error covariance	$M$	Spatial inertia

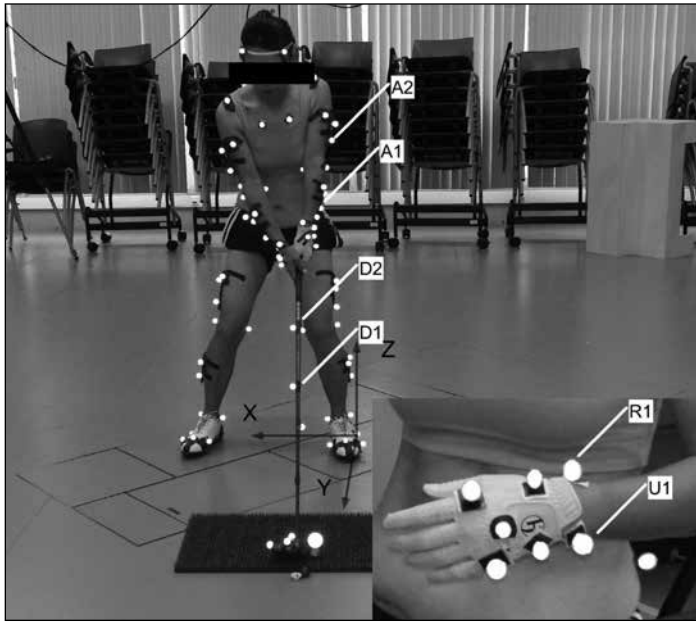
**Design**

Two female golfers with accredited handicaps from the Federation of Portuguese Golfers of 32 (player A, the novice) and 8 (player B, the skilled player) provided their written informed consent to participate in this study. Selected anthropometric measures (Table 2) were obtained using a DASH kit (DASH Schweiz Ltd., Switzerland) and according to published protocols of the International Society for the Advancement of Kinanthropometry (ISAK) (Hume & Marfell-Jones, 2008). Passive markers were attached to the subjects as illustrated in Figure 4. In total, twenty-four reflective markers and four marker clusters were used for the reconstruction of eight body segments (i.e., the trunk, pelvis, thighs, shanks and feet). Marker coordinates for the period between the beginning of the downswing of a golf swing and up to the instant before impact were then acquired. Motion capture was undertaken using an optoelectronic system of twelve Qualisys cameras (type: Oqus-300, Qualisys AB, Sweden) operating at 300Hz. Ground reaction force (GRF) was measured using a Kistler force plate (type: 9865B, Kistler Instruments, Switzerland). Center of Pressure (COP) data were also obtained so as to standardize the same feet locations, over the course of the swings, in both players. The first COP position for each performer’s left foot was used to set the origin of the global frame for that player at the beginning of the downswing (Figure 4).

Our anatomical coordinate (marker based) system was then defined by six landmarks, as follows: the origin was the midpoint of the line joining the distal ulna (U1) and radius (R1); the Z-axis was the line joining D<sub>1</sub> and D<sub>2</sub> of the club shaft; the Y-axis was defined by the cross-product of the Z-axis and the line joining

**Table 2    Characteristics of the Two Female Golf Players Who Participated in the Study.**

Subject	Age (years)	Height (cm)	Mass (kg)	Handicap	Experience (years)	Rounds per Year
A	17	167	54	32	1	10
B	51	165	55	8	15	110



**Figure 4** Position of attached markers on the body of the subject that adopted the stance at the instance of the club addressing the ball. The separate panel depicts individual markers attached on the hand. The labeled markers were used to build an anatomical reference frame within which the grip reference frame was coincident at the beginning of the downswing. The origin of the global frame coincides with the first COP location of the left foot.

$A_1$  and  $A_2$  of the arm; the X-axis is orthogonal to the quasi-sagittal plane defined by the Z- and Y-axes and is also the cross product of two axes (Figure 4). The grip reference frame coincided with the anatomical coordinate system that was established at the initial configuration of the downswing of “the player-club interface”.

Data related to the players’ swings were collected using a driver (R580XD, Taylor Made Golf Company, USA), that had been instrumented with six markers. The club was unknown and unfamiliar to both the novice (A) and the skilled (B) player. In this way, both players started testing under the same control conditions, and a more effective golf swing could be more easily attributed to more highly attuned haptic perception.

The players were instructed to actually hit a golf ball that had been covered in retro-reflective tape, so that the action of their golf swing was focused on hitting and not on “swing technique”.

Subsequent analysis for the player’s six landmarks as well as for the markers on the club was restricted to the downswing phase of the swing i.e., the phase where the wobbling of the left wrist may happen, from the beginning of the downswing to just before impact with the ball. We present results that apply our modeling approach to the inertia tensor that was obtained by geometric scaling (Zatsiorsky, 1998) from a previously published dataset (S. J. MacKenzie, 2005). To be clear, we

did not measure the inertia tensor, but we used data obtained from a club that was made by the same manufacturer. Then, the mass moment of inertia of the club was geometrically scaled by assuming that the moments of inertia of each segment are proportional to the mass times a linear dimension squared (as detailed by Zatsiorsky in 4.4.2 of his book [Zatsiorsky, 1998]). We tuned segment parameters for a sample (for mass, location of center of mass, inertia [S. MacKenzie & Sprigings, 2009a]) to the specific club that was used in our experiment.

The test that we conducted, to compare the perception-action manifolds of the two players of different skill levels, is based on the two ruled surfaces generated in terms of  $e_3$ —the perception of the inertia tensor and ISA—the corresponding actions of the player.

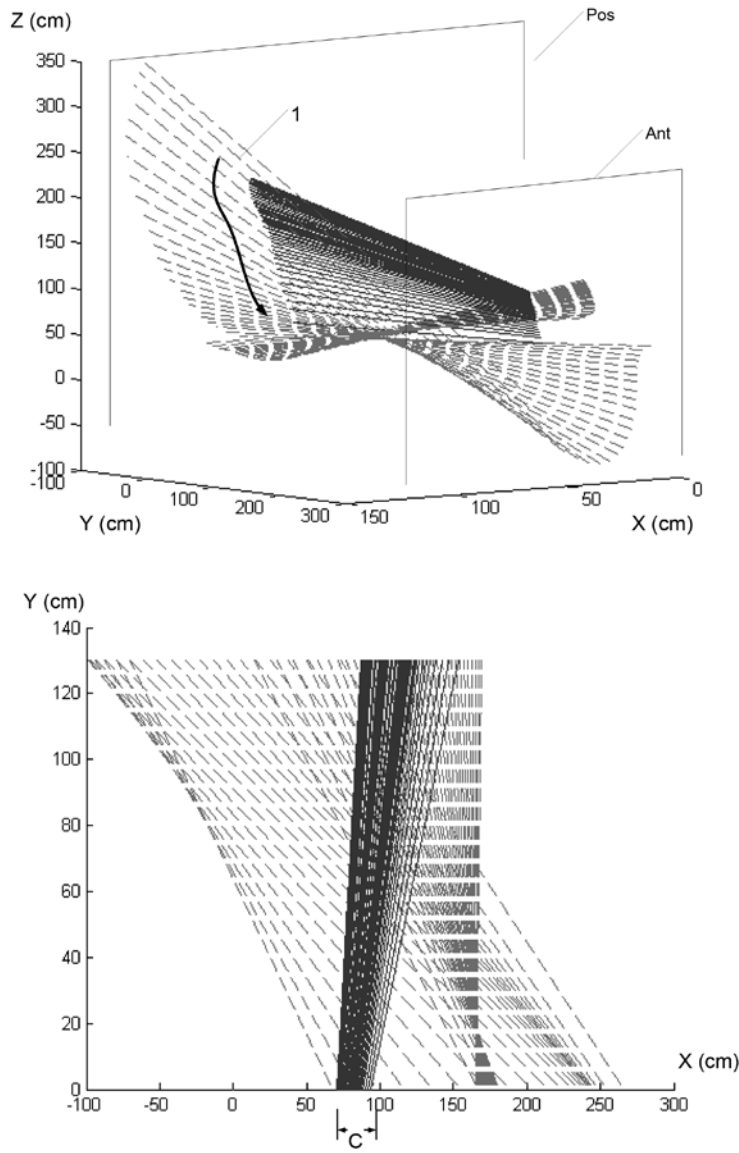
## Results

A difference in the profiles of the perception-action coupling manifold was found to exist between the skilled and the novice player. The two swing patterns produced the typical shape of generator surfaces i.e., small motion steps of the club occurred in the spatiotemporal view as it progressed through the downswing (Figure 5 for player A [the novice] and Figure 6 for the skilled player B) (and in which the ISA are represented as solid lines and  $e_3$  as dashed lines, respectively). In both cases, ISAs are represented relative to the origin of the global frame (Figure 4) which in turn was set at the initial position of the center of pressure of the left foot during the downswing phase. Note that the action pathways of the stroke exhibited a distinctive profile that is characterized by a sharp down turn in the progression of the ISA of the novice (Figure 5 [b]), mixed with the  $e_3$  pathways. In contrast, the location(s) of the ISA of the skilled player (Figure 6 [b]) both exhibited a wider distribution than that of the novice, and lagged behind her  $e_3$  pathways. The locations of them (50 cm [C] moved 95 cm to left of the origin vs. 100 cm [D] moved 170 cm to left of the origin). These differences directly indicate that the skilled player began her swing of the club in alignment with both the migration of the ISA of the club and its  $e_3$  pathways. In other words, her action on the club was continuously phase-matched. Her ISA is assumed to consistently lag behind the  $e_3$ . The novice, however, produced an ISA that displayed multiple departures from phase synchrony.

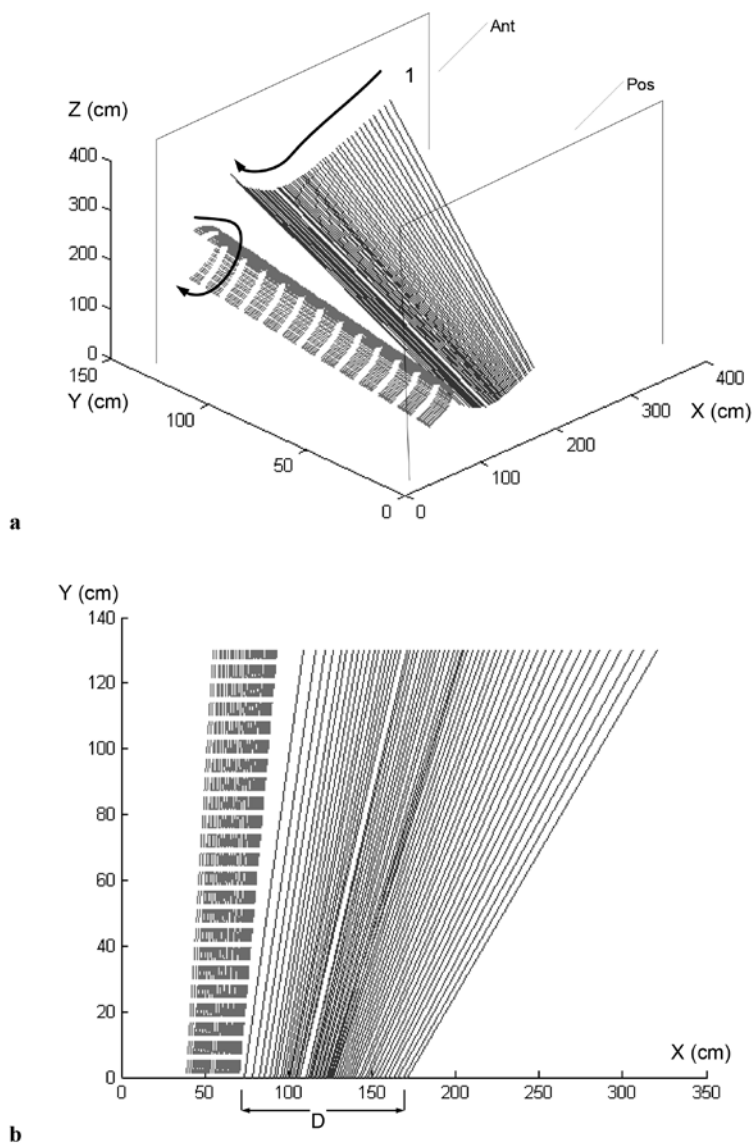
In theoretical terms, a screw surface that in itself is ruled by an ISA is formed to visualize the migration of action. Another screw surface, ruled by  $e_3$ , is formed to visualize the migration of the motion screw. In this way, the characteristics of motion of the swing can be estimated intuitively based on the shape as well as their alignment relative to each other. Our results showed that the form of the  $e_3$  was different between the skilled and in the novice player: Player A, the novice, produced perturbations in  $e_3$  and consequently in ISA, whereas player B, the skilled player, produced an  $e_3$  shape that was uniformly aligned with the path of ISA (Figure 5 [a] and 6 [a]). However, greater congruency was observed in the alignment of the two surfaces in the skilled player compared with the novice. Moreover, by virtue of the positioning of the ISAs alongside the  $e_3$  of the club, the skilled player achieved a motion that the ISAs could be somewhat connected with the principal inertia axes  $e_3$ .

For the novice player, in  $e_3$ , the principal axes were not aligned with ISA's. As such, the instantaneous location of rotational axes showed some perturbation



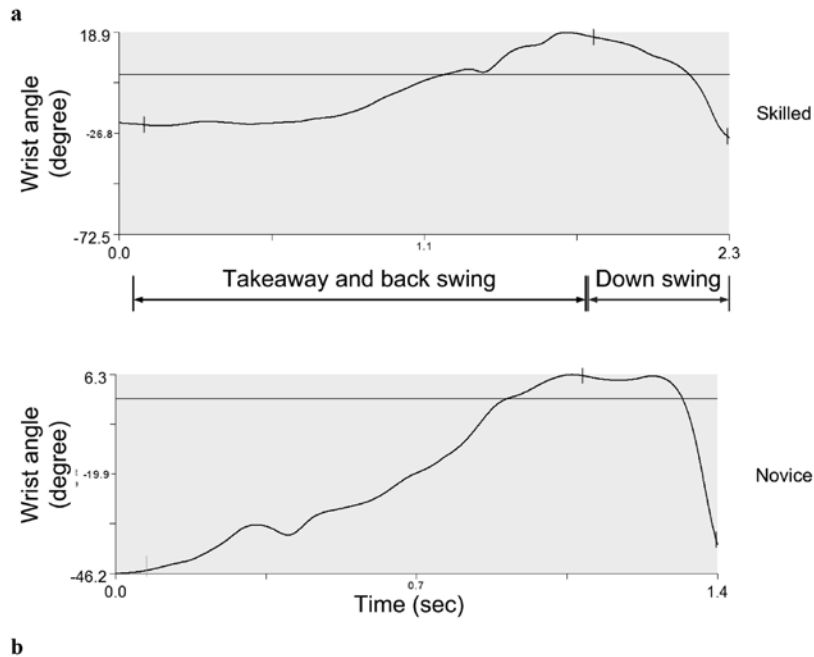


**Figure 5** Player A, the novice golf player, produced a three-dimensional spatiotemporal view on the instantaneous screw axes (ISA, solid lines) and the instantaneous principal axes of inertia ( $e_3$ , dashed lines) for small motion steps (300Hz) that were projected onto the postero-medial side of the player (a) and onto the superior side of the player(b). The end-points of axes are at the intersections with anterior (Ant) and Posterior (Pos) planes, which are located 100 and 300 cm off the origin of the global frame. The first axis, indicated by 1, represents the beginning of the downswing. The arrow indicates where the subsequent axes have migrated at every 0.0333 second of time step (units in cm).



**Figure 6** Player B, the skilled player, produced a three-dimensional spatiotemporal view on the instantaneous screw axes (ISA, solid lines) and the instantaneous principal axes of inertia ( $e_3$ , dashed lines) for small motion step (300Hz) that were projected onto the postero-medial side of the player (a) and onto the superior side of the player (b). In an effort to verify positioning perception and action relation in time-sequence of motion data, the club ISA screws are shown to be regularly projective to the  $e_3$ . This representative analysis indicates a close connection (Spatial-temporal representation of ISA versus the  $e_3$ ) during the down swing for player B.

as the club wobbled. The significant amount of flexion/extension that occurred in the lead wrist of both golfers during the down swing is reported (Figure 7), and hence “flat left wrist” (Hanson, 2007) is not fundamental to swing performance in terms of the “perception- action coupling manifold”. However, the expert kept flat left wrist (Figure 7 [a]) at takeaway contrasting with the novice (Figure 7 [b]) (Leadbetter & Simmons, 2004). The dynamics of the player-club interaction dictate that these events result in changes to the forces that are applied to the golfers by the club. Thus mal-alignment of two screw surfaces is indicative of how a novice player swings the club, in turn leading to the creation of wobbling inertia.



**Figure 7** Player B, the skilled player, kept her left wrist flat at takeaway; however, she may have used extension of the left wrist during the down swing (a). Contrary to what was observed on player B, Player A, the novice golf player produced both excessive wrist motions at takeaway and during the down swing (b) (positive angles indicate flexion and negative angles indicate extension).

## Discussion

The purpose of this study was to obtain a desired perception-action coupling manifold that can be used to explain differences in golf swing performance. We have demonstrated that perception-action coupling manifold, which was generated in terms of two screw axes surfaces during the downswing, is based on natural law—rather than on mental constructs—in agreement with the ecological psychology

literature. In other words, said perception-action coupling manifold is a purely geometric representation of the player-club interaction. Each player has his or her own characteristic interaction traces during the golf swing. This perception-action coupling manifold, which utilizes correlating alignments of both the generating lines of ISA and  $e_3$ , can be used to investigate how a novice can perceive available affordances of making an effective swing (Gibson, 1979).

A promising technique, the ISA theory, could provide a better expression of segment rotation (Vena, Budney, Forest, & Carey, 2011a, 2011b). However, the ISA's of different clubs have not yet been reported and the club itself is not currently considered to have an important influence on the mechanics of the golf swing. We note, however, that in the automobile industry it has been proposed that the ISA surfaces for the car's suspension be included in the performance index of the vehicle's dynamics (U. Lee, 2009). While golf shaft manufacturers have previously used instrumented clubs in an attempt to understand the characteristics of each golfer's swing (N. Lee, Erickson, & Cherveny, 2002), it is still unclear how shaft strain and actual swing performance are interrelated. If the primary means of improving golf swing technique is the repetitive replication of certain trajectories, then paying more attention to the ISAs of the club in holistic terms (as opposed to how the segmental movements are produced, as in traditional golf research (Kelley, 2006)) may be an important avenue for further research. We note that this study appears to be the first to use perception-action coupling manifold to assess differences in the performance of the golf swing between a skilled and a novice player.

When it is presented as an equivalence of error covariance with inertia tensor (Table 2), wobbling inertia is directly related to the perturbation of the error covariance. Perturbation of the error covariance may in itself prove useful as a measure of deviation from established patterns, as we have hypothesized. Wobbling inertia could be one of the factors that explains why more variability in muscle activity and swing time between successive swings exists in novice golfers than in the more skilled player (Barclay & McIlroy, 1994). We have demonstrated that rather than being localized in an internal structure, control of the wobbling inertia is distributed over the golfer-club system (Gibson, 1979). The performer's whole body actions synthesized the club's ISA. The continuous act of perceiving includes ego-perception. The question is, "how do golfers go from the perception of inertias of the club to perception of what they afford?" Golfers perceive the inertias when exploring how they can be used to achieve the goal of hitting the ball. The perception-action coupling manifold could measure the disturbance patterns of player-specific swing as a consequence of a club's wobbling inertia.

This study, however, is limited by the fact that it is a case study. Our work involved insufficient data collection for us to be able to do more than speculate on the implications of our results to entire skill levels. Nor did we include the most critical instance of club to ball impact—and the associated launching parameters of head velocity, launch angle, and backspin— in the modeling process. We therefore suggest several areas for future research: 1) additional testing of golfers of various physical characteristics, handicap levels, and swing styles—so as to obtain data that will allow for better understanding of the interaction between the player and his/her equipment; 2) testing of the characteristics of different clubs in the same player, 3) validation of indoor motion capture systems and the results of laboratory-based findings against actual golf performances and 4) the exploration of how

the technology (including computational functions (MATLAB, Mathworks, Inc., Natick, MA, USA) for ISA and  $e_3$ ) that was used in this study may be transferred into improvement in applied practice –golf research, aiding both golfers and their coaches to optimize performance across a wide range of player swing profiles and handicap levels.

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## Appendix A: Screw Representation of Line- and Free-Vectors

Higher order geometry, involving three dimensional homogeneous Plücker coordinates, was used to define a mapping from lines in three dimensional projective space ( $P^3$ ) to a set of ordered, homogeneous six tuples in six dimensional projective space ( $R^6$ ) (Jacobs et al. 2009; Michaels et al. 2008). It should be noted that the representation of said system of geometrical entities, in this case the lines of  $P^3$ , involves the mapping not on the entire space of  $R^6$  but on a certain algebraic manifold within it. The coordinate system of the screw axis,  $\$$  is then written as:

$$\$ \equiv (L, M, N; P, Q, R) \in R^6 \quad (1)$$

where the variables  $L, M, N$  represent the orientation of the axis of  $\$$ ; and the variables  $P, Q, R$  define the moment vectors of  $\$$ . When normalized by  $L^2 + M^2 + N^2 = 1$  such that  $L = L, M = M$  and  $N = N$ , the two sets of coordinates become related by a scalar multiplier  $\rho$ :

$$\$ \equiv (L, M, N; P, Q, R) \equiv \rho \$ \equiv (\rho L, \rho M, \rho N; \rho P, \rho Q, \rho R) \quad (2)$$

A unit screw (to which no mechanical significance is attached until a twist [wrench] of scalar amplitude [intensity]  $\rho$  is associated with it) and which is considered to be a geometric element, is then defined as in Equation (2). We note that the screw is defined by the relative value of its (homogenous) coordinates, rather than by the absolute value. An infinite pitch screw is defined as a free-vector. Examples are translation and couples. A zero pitch screw is defined as a line-vector. Examples are rotation and forces.

For a line, or a screw of zero pitch, the condition of orthogonality has to be met:

$$\begin{aligned} (L, M, N) \cdot (P, Q, R) &= 0, \\ LP + MQ + NR &= 0 \end{aligned} \quad (3)$$

Such homogeneous coordinates of lines represent those points of five-dimensional projective space that are contained in the Klein quadric (Pottmann et al. 2004).

A useful interchange operator is now introduced (Lipkin and Duffy 1988):

$$[\Delta] \equiv \begin{bmatrix} O & [I_3] \\ [I_3] & O \end{bmatrix} \quad (4)$$

Namely, a six-by-six matrix is defined in which each sub-matrix  $[I_3]$  is the three-by-three identity matrix and the zeros indicate two three-by-three arrays of zeros to fill out the matrix. A line, screw, twist, or wrench 'ray-coordinate' order expressed as a column matrix is converted to 'axis-coordinate' order when it is pre-multiplied by  $[\Delta]$ , such that for a screw:

$$[\Delta][L \ M \ N \ P \ Q \ R]^T = [P \ Q \ R \ L \ M \ N]^T \quad (5)$$

The development of the transformation tensor from noisy data and determination of the ISA has been previously presented (Kiat Teu and Kim 2006).



Two adjacent configuration frames are applied to introduce the screw transformation tensor. All the screws at the reference  $j$  frame can be transformed and expressed in the  $i$  frame by introducing a six-by-six screw transformation tensor.

$$\begin{bmatrix} L_i \\ M_i \\ N_i \\ P_i \\ Q_i \\ R_i \end{bmatrix} = \begin{bmatrix} [R_{ij}] & [0_3] \\ [X_{ij}][R_{ij}] & [R_{ij}] \end{bmatrix} \begin{bmatrix} L_j \\ M_j \\ N_j \\ P_j \\ Q_j \\ R_j \end{bmatrix} \quad (6)$$

or

$$[\$'_i] = [\$_{ij}][\$'_j] \quad (7)$$

Where  $[\$_{ij}]$  is a six-by-six screw transformation tensor. Here,  $[X_{ij}]$  is skew-symmetric and represents points of origin (origin of the displacement) and  $[R_{ij}]$  is the orthogonal rotation tensor.  $[0_3]$  indicates zeros populating the remainder of the transformation tensor.

## Appendix B: The Representation of Spatial Rigid-Body Inertias

In this section of our paper, therefore, we present a method of free-vector decomposition for the spatial rigid-body inertia tensor. The linear map from the spatial acceleration space to the spatial force space was defined as the spatial inertia tensor. This spatial inertia (of a rigid body) has been successfully represented by means of a set of principal screws of inertias (Ciblak 1998). Consider the situation in Figure 1: A club has a mass,  $m$ ; its center of mass,  $C_M$ , is given by the position vector,  $C$ ; and the inertia tensor about its center of mass is  $J$ . The club is at rest, and experiences a force,  $f$ , acting along a line passing through the center of mass, and a couple,  $\tau_C$ . The resulting acceleration is determined by an angular acceleration,  $\alpha$ , along an axis passing through the center of mass ( $C_M$ ), the linear acceleration,  $a_c$ , on,  $C_M$ .

The equation of motion is considered as a mapping from the twist-like screw acceleration to a wrench space (Featherstone 1983).

$$\begin{bmatrix} f \\ \tau_C \end{bmatrix} = \begin{pmatrix} m1 & 0 \\ 0 & J \end{pmatrix} \begin{bmatrix} a_c \\ \alpha \end{bmatrix} \quad (8)$$

where  $m1 = \begin{bmatrix} m & & \\ & m & \\ & & m \end{bmatrix} = M$ , and  $1$  is the  $3 \times 3$  identity matrix.

Since all of the spatial quantities are referred to the center of mass, the linear and angular components of motion are decoupled—the linear acceleration being entirely due to the force, and the angular acceleration being a result of the couple. To transform Equation 8 into the origin of the joint axis (Figure 1), we obtain

$$\begin{bmatrix} f \\ \tau_0 \end{bmatrix} = \begin{pmatrix} m1 & H \\ H^T & I \end{pmatrix} \begin{bmatrix} a_0 \\ \alpha \end{bmatrix} \quad (9)$$

Where  $H = C \times M$ ,  $I = J + C \times M C^T$ , and  $C \times$  is the anti-symmetric skew matrix corresponding to  $C$ .

$$C \times = \begin{pmatrix} 0 & -C_z & C_y \\ C_z & 0 & -C_x \\ -C_y & C_x & 0 \end{pmatrix} \quad (10)$$

Due to its special  $6 \times 6$  form, the spatial inertial tensor

$$M_o = \begin{pmatrix} m1 & H \\ H^T & I \end{pmatrix} \quad (11)$$

is expected to have special eigen structures.

The spatial inertia tensor  $M_o$  represented at the origin is a symmetric, positive definite tensor and transforms to any point “A” by the spatial Jacobian,  $\Phi$  according to

$$M_A = \begin{bmatrix} 1 & 0 \\ -r_{A/O} \times & 1 \end{bmatrix} M_o \begin{bmatrix} 1 & 0 \\ -r_{A/O} \times & 1 \end{bmatrix}^T \quad (12)$$

where  $r_{A/O} = \overrightarrow{OA}$  and  $\Phi = \begin{bmatrix} 1 & 0 \\ -r_{A/O} \times & 1 \end{bmatrix}^T$ .

The eigenvalue problem provides a unique decomposition of  $M_o$  as

$$\begin{pmatrix} m1 & H \\ H^T & J \end{pmatrix} = M_o \begin{bmatrix} f & o \\ \tau & \gamma \end{bmatrix} \begin{bmatrix} m_f & o \\ o & m_\gamma \end{bmatrix} \begin{bmatrix} f & o \\ \tau & \gamma \end{bmatrix}^T \quad (16)$$

Where  $m_f$  and  $m_\gamma$  are representing the eigen values of mass and mass moment of inertia respectively (following the common notational tactics for the principal axes of inertia, we use  $e_1, e_2, e_3$  for  $m_\gamma$  and, to the corresponding principal moment of inertia,  $I_1 \leq I_2 \leq I_3$  for  $\gamma$ ).

One might wonder whether the decomposition based on the solution for the free-vector eigenvalue problem would be different at another point A. We apply the transformation rule (Equation 12) to the above decomposition (Equation 13).

$$= \begin{bmatrix} 1 & 0 \\ -r_{A/O} \times & 1 \end{bmatrix} \begin{bmatrix} f & o \\ \tau_o & \gamma \end{bmatrix} \begin{bmatrix} m_f & 0 \\ 0 & m_\gamma \end{bmatrix} \begin{bmatrix} f & o \\ \tau_o & \gamma \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ -r_{A/O} \times & 1 \end{bmatrix}^T \quad (14)$$

$$= \begin{bmatrix} f & 0 \\ \tau_o - r_{A/O} \times f & \gamma \end{bmatrix} \begin{bmatrix} m_f & 0 \\ 0 & m_\gamma \end{bmatrix} \begin{bmatrix} f & 0 \\ \tau_o - r_{A/O} \times f & \gamma \end{bmatrix}^T \quad (15)$$

$$= \begin{bmatrix} f & 0 \\ \tau_A & \gamma \end{bmatrix} \begin{bmatrix} m_f & 0 \\ 0 & m_\gamma \end{bmatrix} \begin{bmatrix} f & 0 \\ \tau_A & \gamma \end{bmatrix}^T \quad (16)$$

which shows that  $M_A$  is decomposed by the same eigenscrews in the same manner, just represented at A. That is, no matter where the problem is posed the same eigenscrews form the basis of the decomposition.

## Eigen Structure of Spatial Inertia Tensor at Center of Mass

We considered the free-vector eigenvalue problem: Determine a twist that only causes a parallel couple (a free-vector):

$$\begin{pmatrix} m1 & H \\ H^T & J \end{pmatrix} \begin{bmatrix} \delta \\ \gamma \end{bmatrix} = m_\gamma \begin{bmatrix} 0 \\ \gamma \end{bmatrix} \quad (17)$$

For the inertial tensor at  $C_M$  (Figure 2), the eigenvalue problem is:

$$\begin{pmatrix} m\mathbf{1} & 0 \\ 0 & \mathbf{J} \end{pmatrix} \begin{bmatrix} \delta \\ \gamma \end{bmatrix} = m_\gamma \begin{bmatrix} 0 \\ \gamma \end{bmatrix} \quad (18)$$

It is seen from these equations that  $m_\gamma$ ,  $\gamma$  are the eigenvalues and vectors of  $\mathbf{J}$ , and  $\delta=0$ .

We also considered the different eigenvalue problem: Determine a wrench that only causes a parallel translation (a free-vector):

$$\begin{pmatrix} m^{-1}\mathbf{1} & 0 \\ 0 & \mathbf{J}^{-1} \end{pmatrix} \begin{bmatrix} \mathbf{f} \\ \tau \end{bmatrix} = m_f^{-1} \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix} \quad (19)$$

where, the  $m_f^{-1}$  is usually termed the mobility or admittance, which is the inverse of the spatial inertia tensor. It is apparent from these equation that  $m_f = m$ ,  $\mathbf{f}$  is arbitrary, and  $\tau=0$ , i.e., the eigentwist and eigenwrench are pure rotation and pure forces through  $C_M$ . Eigen structure of the inertial tensor at  $C_M$  (Figure 2) shows the set of all lines, generated through a point  $C_M$ , and what is referred as the force bundle. Note that the twist in Equation (18) that causes only a pure couple parallel to the rotation part is called an eigentwist and the wrench in Equation (19) that causes only a pure translation parallel to the force part is called an eigenwrench.

## Eigen Structure of Spatial Inertia Tensor Away From Center of Mass

It is not clear what the precise influence of the inertia tensor is on the relevant proprioceptive mechanical variables of the “dynamic touch”, (which may be computed through separation of pure rotations and pure translation) (Menger and Withagen 2009). The moment of inertia ought to be computed with respect to the wrist, the presumed point of rotation (Fitzpatrick et al. 1994; Solomon and Turvey 1988). On the other hand, the end of the rod ought to be taken as the reference point (Withagen and Michaels 2005).

We characterized the eigen structure by clarifying how its structure is related to an arbitrary point, i.e., the location of the wrist joint axis. The eigenvectors at points away from the center of mass have distinct properties. The line-vector eigenvalue problems are different at different points. This leads to the previous well-known decomposition of the inertial tensor into their geometric and constitutive contents. A new eigenvalue problem is obtained and shown to be complementary to the free-vector eigenvalue problem. The new eigenvalue problem is not unique since every point in space generates a distinct line-vector subspace. The properties of the line-vector eigenvalue problem at points other than  $C_M$  are distinct.

Consider a point at joint axis,  $A \neq C_M$  whose position from  $A$  to  $C_M$  is  $\mathbf{C}$ . The line-vector (pure force) eigentwist subspace of the inertia tensor at  $A$  is formed by one pure translation parallel to  $\overline{AC_M}$  and two pure rotations which intersect the line through  $A$  and  $C_M$  (Figure 3). If  $A$  is on a principal inertia axis then the line-vector eigentwist are parallel to the principal inertia directions.

$$\begin{pmatrix} m^{-1}\mathbf{1} - \mathbf{C} \times \mathbf{J}^{-1} \mathbf{C} \times & -\mathbf{C} \times \mathbf{J}^{-1} \\ \mathbf{J}^{-1} \mathbf{C} \times & \mathbf{J}^{-1} \end{pmatrix} \begin{bmatrix} t_A \\ 0 \end{bmatrix} = m_A^{-1} \begin{bmatrix} t_A \\ W_A \end{bmatrix} \quad (20)$$

Where  $m_A^{-1}$  and  $t_A$  are the classical eigenvalue and eigenvector of the matrix  $m^{-1}I - C \times J^{-1} C \times$ . The symmetry of  $m^{-1}I - C \times J^{-1} C \times$  ensures the existence of three real eigenvalues and eigenvectors of  $t_A$ . Although the set of vector  $t_A$  are orthogonal, the sets obtained by  $-C \times J^{-1} t_A = m_A^{-1} W_A$  are not. Thus, generally contrary to the free-vector problem, the line-vector eigenscrew does not have orthogonal directions.

The line vector (pure rotation) eigenwrench subspace of the inertia tensor at A is formed by

$$\begin{pmatrix} mI & mC \times \\ -mC \times & J - mC \times C \times \end{pmatrix} \begin{bmatrix} 0 \\ s_A \end{bmatrix} = m_A \begin{bmatrix} n_A \\ s_A \end{bmatrix} \quad (21)$$

where it is seen that  $m_A$  and  $S_A$  are the classical eigenvalue and eigenvector of the matrix  $J - mC \times C \times$ . Although the set of vector  $S_A$  is orthogonal, the sets obtained by  $mC \times S_A = m_A n_A$  are not.

If A is not along a principal inertia axis, the three line-vector (rotations) and eigen wrenches (pure forces) are perpendicular to  $\overline{AC_M}$ . If A is along a principal inertia axis, one free-vector eigenwrench (pure couple) is parallel to  $\overline{AC_M}$  and two pure forces perpendicularly intersect  $\overline{AC_M}$  in principal inertia direction.

## Appendix C: The Haptic System and Dynamic Touch

Dynamic touch is the label given to a particular kind of inquiry into the non-visual perception of spatial and other properties of grasped and manually wielded rigid objects that involves a non-spatial input from muscles and tendons (Turvey, 1998). Successful use of a hand-held tool requires the exertion of muscular force to overcome the inherent translational and rotational inertia of the system (Wagman & Carello, 2001). Doing so requires detection of information relevant to the control of that system—that is, how much force is necessary and how it should be directed (Shockley, Grocki, Carello, & Turvey, 2001). A description of the forces that are required to manipulate an object via dynamic touch is provided by the inertia tensor ( $I_{ij}$ )—a 3x3 matrix that describes the resistance, in different directions, to rotational acceleration about a rotation point in the wrist of the performer. In short,  $I_{ij}$  reflects the mass distribution of the hand-object system in question. The eigenvectors ( $e_1, e_2, e_3$ ), of the hand-plus-tool system describe the orientation of the mass distribution of that system with respect to rotational axes in the wrist. The eigenvalues ( $I_1, I_2, I_3$ , where  $I_1 \geq I_2 \geq I_3$ ) refer to the resistances to rotational acceleration about each of those axes (Craig & Bourdin, 2002; Riley, Shaw, & Pagano, 2005).

Research has suggested that perceived properties of hand-held objects are constrained by  $I_{ij}$  in a lawful and predictable manner. Perceived magnitudes (e.g., length, width) are constrained by the eigenvalues (i.e. perceived length is constrained by  $I_1$ , and perceived width is constrained by  $I_3$ ). Perceived directions (e.g., the orientation of an object in then hand) are tied to the eigenvectors (Turvey, 1996). Furthermore, perception of heaviness is largely constrained by two “higher-order” inertial variables: Volume [ $V = 4\pi/3 (I_1 \times I_2 \times I_3)^{-1/2}$ ] and Symmetry [ $S = 2 \times (I_3 / (I_1 + I_2))$ ], which quantify the mean level of force that is required to rotate the hand-object system, and how those forces should be directed, respectively (Shockley et al., 2001). For example, research has shown that the position perception of whether it is appropriate to displace a given hand-held object is negatively correlated with a combination of  $V$  and  $e_3$  (i.e., objects that were easier to control are perceived as being easier to displace, (Wagman & Carello, 2001). This program of work (Carello & Turvey, 2004), suggests that higher-order inertial variables that are derived from  $I_{ij}$  (in particular,  $V$ ,  $S$ , and  $e_k$ ) are relevant to perception of the properties of hand-held tools. Importantly, such inertial variables are relevant to the understanding of how and where a tool-user needs to grasp a tool, in order to exert task specific control over that tool. This is precisely the goal of this study- to investigate how a performer might controls the full-body action of a golf swing through the perception of the inertia of the club when said club is being acted upon by him/her.

Performers grasp objects, such as golf clubs, in such a way as to configure the higher-order inertial properties of the hands-club system to be appropriate for the task at hand. For example, when grasping an object so as to perform a task that requires power (such as the swinging of a golf club to hit a ball), skilled performers grasp so as to preserve  $V$  and  $e_k$  whilst reducing  $S$ . This creates an asymmetric and elongated mass distribution, and serves to maximize the potential force that is delivered per swing with said club. In short, skilled performers seem attuned to the fact that how an object can be moved determines its potential uses. In grasping that object, they create a function for the hand-plus-object system (Wagman & Carello, 2003).

(Bernstein's (1967) solution to the problem of how to control the motor degrees of freedom of a given system, is that a system with many degrees of freedom can act as if it were a simple system when and only when sufficient constraints have been established between its components. This is done through the coupling of said components in a synergistic manner (Araújo, Davids, & Hristovski, 2006; R. Shaw & Kinsella-Shaw, 1988; R. E. Shaw & Turvey, 1999). However, if the degrees of freedom are cast in terms of efference, then the degrees of constraint that render them controllable and adaptive must be cast in terms of afference. But afference is no less many-dimensional than efference, and no less in need of a principled means of reduction of dimensionality (Turvey, 2007). Defining and understanding the notion of synergy may require an ability to express afference and efference, or more generally perception and action, as concurrently continuous. The central idea is that the afferent and efferent flows that comprise a given synergy are embedded in some measurable quantity that in itself expresses coherent link(s) between the parts and processes that make up said synergy (Turvey, 2007).